Laser Cooling of Atoms: a New Thermodynamic Approach based on Increasing Purity

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We present a new technique to study atomic beam cooling by analyzing the purity of the density matrix. A theory is presented for two-level atoms, both for the case of running waves and the case of standing waves. Our approach allows us to consider the cooling process in terms of thermodynamic laws, based on the definition of cooling in terms of the increase of the purity $Tr(\rho^2)$ in time [1,2]. $Tr(\rho^2)$ has the interpretation of phase-space density. For an incoherent Gaussian distribution, increasing $Tr(\rho^2)$ corresponds to decreasing the width of the distribution in position and momentum, but more generally quantum coherence is also taken into account. This definition of cooling eliminates the need for the problematic concept of temperature when dealing with non-equilibrium systems, and is therefore appropriate for characterizing distributions such as those obtained in a cold atom beam by a Zeeman-slower apparatus. Note that at present the Zeeman slowing technique allows the production of a continuous flux of slow and cold atoms with highest atom phase-space density available [3].

Let us consider the slowing of an atomic beam by one running wave. We start with the equations for the elements of the matrix density of two-level atom in the Wigner representation as

$$\frac{d}{dt}\rho_{22}(\vec{r},\vec{p},t) = g_{21}\rho_{12}(\vec{r},\vec{p}-\hbar\vec{k}/2,t)\exp(i\vec{kr}) - g_{12}\rho_{21}(\vec{r},\vec{p}+\hbar\vec{k}/2,t)\exp(-i\vec{kr}) - 2i\gamma\rho_{22}(\vec{r},\vec{p},t),$$

$$\frac{d}{dt}\rho_{21}(\vec{r},\vec{p},t) = g_{12}[\rho_{11}(\vec{r},\vec{p}-\hbar\vec{k}/2,t) - \rho_{22}(\vec{r},\vec{p}+\hbar\vec{k}/2,t)]\exp(i\vec{kr}) - i\gamma\rho_{21}(\vec{r},\vec{p},t), \quad (1)$$

$$\frac{d}{dt}\rho_{11}(\vec{r},\vec{p},t) = -g_{21}\rho_{12}(\vec{r},\vec{p}+\hbar\vec{k}/2,t)\exp(i\vec{kr}) + g_{12}\rho_{21}(\vec{r},\vec{p}-\hbar\vec{k}/2,t)\exp(-i\vec{kr})$$

$$+2i\gamma\int d\vec{n}\Phi(\vec{n})\rho_{22}(\vec{r},\vec{p}+\hbar k_0\vec{n},t),$$

where $\frac{d}{dt} = \frac{\partial}{\partial t} + \vec{v}\frac{\partial}{\partial \vec{r}}$, $\vec{v}$ is the velocity of the atom, and the function $\Phi(\vec{n})$ determines the relative probability of spontaneous emission of a photon in the direction of the unit vector $\vec{n}$.

The definition of the purity (or Renyi entropy) is [1]:

$$P(t) = \int_{\infty}^{\infty} Tr(\rho(\vec{r},\vec{p},t)^2)\, d\vec{r}d\vec{p}, \quad (2)$$

where $Tr(\rho^2) = \rho_{11}^2 + \rho_{22}^2 + 2|\rho_{12}|^2$ and $\rho_{i,j}(\vec{r},\vec{p},t)$ are the elements $\rho_{i,j}(\vec{r},\vec{p},t)$ from (1). To obtain a master equation for $Tr(\rho^2)$, we expand the density matrix elements in (1) in a power series around the point $\vec{p}$, assuming the recoil energy R of the atom is much smaller than the energy of an optical transition: $R = \hbar^2k^2/(2M) << \hbar\gamma$. Using the steady state solution for the density matrix elements to first order in $\varepsilon = R/\hbar\gamma$, we obtain the following equation for the time evolution of the velocity distribution function $w = \rho_{11} + \rho_{22}$:

$$\frac{d}{dt}w = -\hbar k\gamma\frac{\partial}{\partial p_z}(f_0 w), \qquad f_0 = \frac{2\Omega^2/\gamma^2}{1 + (\Delta/\gamma + kv/\gamma)^2 + 2\Omega^2/\gamma^2}, \tag{3}$$

where f_0 is the light force, Ω is the Rabi frequency, Δ is the detuning of the laser frequency from the optical transition, and γ is the rate of spontaneous decay. To the same order in ε, the master equation for $Tr(\rho^2)$ is given by

$$\frac{d}{dt}Tr(\rho^2) = -\hbar k\gamma\frac{\partial}{\partial p_z}(f_0 w^2) + \hbar^2k^2\gamma[(1 + \frac{1 + (\Delta/\gamma + kv/\gamma)^2 f_0}{2\Omega^2/\gamma^2})w\frac{\partial^2}{\partial p_z^2}(f_0 w)]. \tag{4}$$

Consider now the short time evolution of $Tr(\rho^2)$. In this case, we can neglect the second term on the right side of (4) and obtain the solution of the simplified equation as

$$Tr(\rho^2) = Tr(\rho(v;0)^2)\exp[\alpha\tau],$$

where

$$\alpha = 8\frac{[(\Delta + kv)/\gamma]}{2\Omega^2/\gamma^2}\frac{f_0^2}{f_0^2 + 2}, \qquad \tau = \frac{\hbar k^2}{2M}t, \tag{5}$$

and $Tr(\rho^2(v;0)) = w^2(v;t=0)$. Assuming now that the initial velocity distribution function is characterized by a thermal distribution, the purity of the two level-atom beam in the field of the running wave can be represented by eq. (2) as

$$P(t) = N(8/\pi)^{1/2}\frac{\varepsilon}{(\delta v)^2}\int_0^\infty \exp[-\frac{(v - v_0)^2}{(\delta v)^2}]\exp[\alpha\tau]\,dv, \tag{6}$$

where

$$\alpha = 4\frac{(2\Omega^2/\gamma^2)(\Delta/\gamma + kv/\gamma)}{(1 + (\Delta/\gamma + kv/\gamma)^2)^2} \quad \text{for} \quad \Omega^2/\gamma^2 << 1;$$

$$\alpha = \frac{8}{3}\frac{(\Delta/\gamma + kv/\gamma)}{2\Omega^2/\gamma^2} \quad \text{for} \quad 2\Omega^2/\gamma^2 >> |\Delta|/\gamma.$$

As one can see from eq. (6), the purity is increasing in time for the case of both small and large saturations of the atomic transition. The numerical estimation shows that for the values $2\Omega^2/\gamma^2 = 10, \Delta/\gamma = -70$, which are typical for the deceleration of an atomic beam, the purity reaches unity for interaction time $t \approx 50(\hbar k^2/2M)^{-1}$. In the evolution of an atomic beam, this time corresponds to the formation time of the narrow peak in the velocity distribution.

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